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# On the nucleon self-energy in nuclear matter

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**Abstract.** We consider the nucleon self-energy in nuclear matter in the absence of Pauli blocking. It is evaluated using the partial-wave analysis of *NN* scattering data. Our results are compared with that of a realistic calculation to estimate the effect of this blocking. It is also possible to use our results as a check on the realistic calculations.

**PACS.** 21.30.-x Nuclear forces – 21.65.+f Nuclear matter

## 1 Introduction

A fundamental problem of nuclear physics is to explain the properties of nuclear matter (and finite nuclei) in terms of an effective field theory at low energy based only on the (chiral) symmetry of QCD. While such a theory has been eminently successful for systems like $\pi\pi$ and $\pi N$ [1], a satisfactory theory for the *NN* system has been difficult to formulate due to the presence of two-nucleon bound or virtual states close to the threshold of *NN* scattering [2]. In particular, the leading chiral four-nucleon interaction predicts an absurdly large value for the self-energy of the nucleon at normal nuclear density [3].

There is, however, a semi-phenomenological approach that yields fairly accurate values for different observables in nuclear matter. Here the *NN* potential is constructed by exchanging low-mass bosons in the $t$-channel [4]. The coupling and other parameters in the potential are determined by experimental data on the deuteron and the low-energy *NN* scattering. The dynamics is formulated on the basis of a relativistic version of the Brueckner-Hartree-Fock method [5,6], where the reaction matrix satisfies a three-dimensionally reduced Bethe-Salpeter equation in nuclear medium. The Dirac equation for the nucleon incorporates the scalar part of the self-energy due to its interaction with nucleons in the medium. The self-energy itself is given by the diagonal element of the reaction matrix. The system of equations is then solved self-consistently.

In this work we study the nucleon self-energy in a certain theoretical limit. We observe that if we suspend the Pauli-blocking operator (projecting onto the unoccupied states) in the equation for the reaction matrix, it coincides with the one for the scattering matrix in vacuum. Further, if we do not include the relevant part of the single-particle self-energy in the mass term in the Dirac equation, the requirement of self-consistency does not arise any more. In this limit the self-energy is given by an integral over the spin-averaged, forward *NN* scattering amplitude in vacuum, which can be evaluated entirely with the experimental data.

There is a well-known expansion in statistical mechanics, called the virial expansion, whose first term for the in-medium self-energy would give exactly the theoretical limit considered above [7–9]. Here we employ this method to derive the formula for the limiting self-energy of the nucleon. We then evaluate it at different nuclear densities, using the phase shift analysis of *NN* scattering data, independently of any *NN* potential. It is then compared with the realistic calculation [6] to assess the importance of the effect of Pauli blocking in nuclear matter.

Our calculation would also serve as an important check on the realistic calculation. One has just to repeat the calculation of the self-energy in the original framework itself, using the phenomenological potential and the physical nucleon mass, but in the theoretical limit of omitting the Pauli-blocking operator in the reaction matrix. This result as a function of the nuclear density must agree with that of the present calculation. This, in turn, would confirm our assertion that it is indeed the Pauli-blocking effect which distinguishes the realistic calculation from the one presented here.

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## 2 Derivation of self-energy formula

Here we obtain the leading term in the virial expansion for the nucleon self-energy in nuclear medium [7–9]. We begin by stating clearly the normalization of different quantities. Omitting the nucleon isospin index [10], we take the creation and the annihilation operators for the nucleon with momentum $\boldsymbol{p}$ and spin projection $\sigma\,(=\pm\frac{1}{2})$ to satisfy the anticommutation relation,

$$\{b(\boldsymbol{p},\sigma),b^\dagger(\boldsymbol{p}',\sigma')\}=(2\pi)^3\,2E_p\,\delta(\boldsymbol{p}-\boldsymbol{p}')\delta_{\sigma\sigma'}\,,\qquad E_p=\sqrt{\boldsymbol{p}^2+m^2}\,. \tag{2.1}$$

The single-nucleon state is defined as $|\boldsymbol{p},\sigma\rangle=b^\dagger(\boldsymbol{p},\sigma)|0\rangle$. Similarly the two-nucleon state is $|\boldsymbol{p}_1,\sigma_1;\boldsymbol{p}_2,\sigma_2\rangle = b^\dagger(\boldsymbol{p}_1,\sigma_1)b^\dagger(\boldsymbol{p}_2,\sigma_2)|0\rangle$. Clearly their normalization is fixed by the anticommutation rule (2.1). The (positive-energy) Dirac spinors are normalized such that the spin sum over these spinors, to be used below, is given by

$$\sum_\sigma u(\boldsymbol{p},\sigma)\overline{u}(\boldsymbol{p},\sigma)=\not{p}+m\ .$$

The derivation starts by considering the nucleon self-energy in *vacuum*, which may be expressed as an $S$-matrix element,

$$-i(2\pi)^4\delta^4(p_1'-p_1)\overline{u}(\boldsymbol{p}_1',\sigma_1')\Sigma^{(0)}(p)u(\boldsymbol{p}_1,\sigma_1)=\langle 0|b(\boldsymbol{p}_1',\sigma_1')\,(S-1)\,b^\dagger(\boldsymbol{p}_1,\sigma_1)|0\rangle\ , \tag{2.2}$$

where $S$ is the familiar scattering matrix operator, $S=Te^{i\int\mathcal{L}_{\rm int}(x)\,{\rm d}^4x}$, for an interaction Lagrangian $\mathcal{L}_{\rm int}$. The subscript 1 on the variables of the particle anticipates a second particle in the medium with which the first one will interact. In fact, we shall express below the nucleon self-energy in *nuclear medium* in terms of the *NN* scattering amplitude in *vacuum*, defined, as usual, by

$$\begin{aligned}\langle\boldsymbol{p}_1',\sigma_1';\boldsymbol{p}_2',\sigma_2'|S-1|\boldsymbol{p}_1,\sigma_1;\boldsymbol{p}_2,\sigma_2\rangle=&\\ i(2\pi)^4\delta^4(p_1'+p_2'-p_1-p_2)&\\ \times M(p_1,\sigma_1;\,p_2,\sigma_2\to p_1',\sigma_1',;p_2',\sigma_2')\,,&\end{aligned} \tag{2.3}$$

where $M$ stands for the scattering matrix sandwiched between spinors corresponding to the final and the initial states of the two nucleons.

Below we shall meet the spin-averaged amplitude in the forward direction,

$$\overline{M}(p_1,p_2\to p_1,p_2))=\frac{1}{4}\sum_{\sigma_1,\sigma_2}M(p_1,\sigma_1;p_2,\sigma_2\to p_1,\sigma_1;p_2,\sigma_2), \tag{2.4}$$

With our normalization of states, the amplitude $\overline{M}$ is Lorentz invariant.

To obtain the self-energy in nuclear medium, we have to replace the vacuum expectation value in eq. (2.2) by an appropriate one. Although we specialize later to zero temperature, we take here the most general average over an ensemble of systems maintained at temperature $T\,(=1/\beta)$ with nucleon chemical potential $\mu$. Thus the in-medium self-energy $\Sigma$ is given by

$$-i(2\pi)^4\delta^4(p_1'-p_1)\overline{u}(\boldsymbol{p}_1',\sigma_1')\Sigma(p)u(\boldsymbol{p}_1,\sigma_1)=\langle b(\boldsymbol{p}_1',\sigma_1')(S-1)b(\boldsymbol{p}_1,\sigma_1)\rangle\ , \tag{2.5}$$

where for any operator $O$,

$$\langle O\rangle = Tr[e^{-\beta(H-\mu\mathcal{N})}O]/Tre^{-\beta(H-\mu\mathcal{N})}\ .$$

Here $H$ is the Hamiltonian and $\mathcal{N}$ the nucleon number density operator. Clearly this form of the Boltzmann weight breaks explicit Lorentz invariance and singles out the rest frame of the medium [11].

We now make use of the virial expansion to first order in density. For an operator $O$, the ensemble average in nuclear medium can be expanded as

$$\langle O\rangle=\langle 0|O|0\rangle+\sum_{\sigma_2}\int\frac{{\rm d}^3p_2}{(2\pi)^3 2E_{p_2}}n(E_{p_2})\langle\boldsymbol{p}_2,\sigma_2|O|\boldsymbol{p}_2,\sigma_2\rangle\ ,$$

where $n(E_p)$ is the nucleon distribution function, $n(E_p)=1/[e^{\beta(E_p-\mu)}+1]$. Applying it to the left-hand side of eq. (2.5), we get for the difference $\Sigma^{(n)}(p)=\Sigma(p)-\Sigma^{(0)}(p)$,

$$\begin{aligned}-i(2\pi)^4\delta^4(p_1'-p_1)\overline{u}(\boldsymbol{p}_1',\sigma_1')\Sigma^{(n)}(p_1)u(\boldsymbol{p}_1,\sigma_1)=&\\ \sum_{\sigma_2}\int\frac{{\rm d}^3p_2}{(2\pi)^3 2E_{p_2}}n(E_{p_2})&\\ \times\langle\boldsymbol{p}_2,\sigma_2|b(\boldsymbol{p}_1',\sigma_1')(S-1)b^\dagger(\boldsymbol{p}_1,\sigma_1)|\boldsymbol{p}_2,\sigma_2\rangle\ .&\end{aligned} \tag{2.6}$$

The matrix element in eq. (2.6) will be immediately recognised to be the *NN* scattering amplitude defined above by eq. (2.3). Cancelling the $\delta$-function on both sides, we set $\sigma_1'=\sigma_1$ and sum over $\sigma_1$ also to get

$$-{\rm tr}\{\Sigma^{(n)}(p_1)(\not{p}_1+m)\}=4\int\frac{{\rm d}^3p_2}{(2\pi)^3 2E_{p_2}}n(E_{p_2})\overline{M}(p_1,p_2\to p_1,p_2), \tag{2.7}$$

where the tr(ace) is over matrices in Dirac space. Note the similarity of this equation with the corresponding one in Brueckner theory [12]. There is, however, one important difference: Our first-order formula has the scattering amplitude *in vacuum*, while it is the amplitude *in medium* that enters the equation in Brueckner theory. We shall discuss this point again in sect. 4.

So far we did not state explicitly the isospin structure of the amplitude $\overline{M}$, which is now easy to figure out. We consider symmetric nuclear matter and work in the limit of isospin symmetry. Let the traversing nucleon be in any one of its isospin states, say a proton. It may scatter with a proton or a neutron in the medium. The amplitude is therefore given by the sum,

$$\overline{M}=\overline{M}_{pp\to pp}+\overline{M}_{pn\to pn}\ . \tag{2.8}$$

We now restrict to the case, where the three-momentum $\boldsymbol{p}_1$ is set equal to zero. Then the rest frame of the medium coincides with the lab frame of the scattering process. The self-energy in this frame has the simple Dirac matrix structure,

$$\Sigma^{(n)} = U \cdot 1 + V\gamma^0, \tag{2.9}$$

where the coefficients $U$ and $V$ depend only on the nucleon density. Then the left-hand side of eq. (2.7) simplifies to $-4m(U+V)$. On the other hand, the nucleon propagator with self-energy correction, $i/\{\not p_1 - m - \Sigma^{(n)}(p_1)\}$, reduces, for $\boldsymbol{p}_1 = 0$, to

$$\frac{i}{p_0 - (m+U+V)}\frac{1}{2}(1+\gamma^0)\ , \tag{2.10}$$

in the vicinity of the pole. The shifted pole position is thus given by

$$m^* - \frac{i}{2}\gamma = m+U+V = m - \frac{1}{m}\int \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_{p_2}} n(p_2)\overline{M}(p_2), \tag{2.11}$$

where $m^*$ is the effective mass of the nucleon and $\gamma$ gives the damping rate of nucleonic excitations.

## 3 Evaluation

In our evaluation we restrict ourselves to nuclear matter at zero temperature. In this limit we assume the nuclear medium to be a non-interacting Fermi gas with all states filled up to the Fermi momentum $p_{\mathrm{F}}$, so that $n(\boldsymbol{p}) \to \theta(p_{\mathrm{F}} - |\boldsymbol{p}|)$. For the symmetric medium the number density is then given by

$$\overline{n} = 4\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\theta(p_{\mathrm{F}} - |\boldsymbol{p}|) = \frac{2p_{\mathrm{F}}^3}{3\pi^2}\ ,$$

where $p_{\mathrm{F}}$ is related to the chemical potential by $p_{\mathrm{F}} = \sqrt{\mu^2 - m^2}$.

The scattering amplitudes are generally analysed in the center-of-mass (c.m.) frame, where they are normalized in a slightly different way to get a simple expression for the differential cross-section. One defines a scattering amplitude $f$ related to $M$ by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{|M|^2}{(8\pi E)^2} \equiv |f|^2\,, \tag{3.1}$$

where $E$ is the total energy in the c.m. frame. The results of partial-wave analysis are generally given as functions of the lab kinetic energy $T(=\sqrt{p_2^2 + m^2} - m)$, in terms of which we have $E = \sqrt{2m(2m+T)}$. We also note here the expression for the c.m. momentum in terms of $T$, $k = \sqrt{mT/2}$.

We can now write eqs. (2.11) as

$$m^* - \frac{i}{2}\gamma = m - \frac{2}{\pi}\sqrt{\frac{2}{m}}\int_0^{T_{\mathrm{F}}} \mathrm{d}T\sqrt{T}(2m+T)\overline{f}(T)\ , \tag{3.2}$$

where $T_{\mathrm{F}}$, the upper limit of the integral, is related to $p_{\mathrm{F}}$ by $p_{\mathrm{F}} = \sqrt{T_{\mathrm{F}}(2m+T_{\mathrm{F}})}$ and

$$\overline{f} = \overline{f}_{pp\to pp} + \overline{f}_{pn\to pn},$$

the bar indicating spin averaging as in eq. (2.4). In terms of amplitudes with definite isospin,

$$\overline{f} = 3/2\overline{f}^{(I=1)} + 1/2\overline{f}^{(I=0)}.$$

The full scattering amplitudes are expanded in a series of partial-wave amplitudes, which may then be determined by fitting with experimental scattering data. For the spin-averaged, forward scattering amplitudes, this expansion takes a particularly simple form [13,14],

$$\overline{f}^I(E) = 2\cdot\frac{1}{4}\sum_{jsl}(2j+1)f_{l,l}^{Ijs}(E)\,. \tag{3.3}$$

Here the factor of 2 takes into account the identity of the two nucleons in the scattering process. The total angular momentum $j$ is obtained by coupling the total spin and orbital angular momenta $s$ and $l$, respectively. The Pauli principle restricts the possible amplitudes by requiring the quantum numbers to satisfy

$$(-1)^l(-1)^{1-s}(-1)^{1-I} = -1\,.$$

In general, the amplitude $f^{Ijs}(E)$ is a matrix in the $l$ space, whose diagonal elements enter the sum in eq. (3.3). Below we shall remove the superscripts $I$ and $s$ on the partial-wave amplitudes.

The form of the partial-wave amplitudes is determined by the unitarity of the $S$-matrix. Thus, for the uncoupled waves, $s=0, l=j$ and $s=1, l=j$, we have the single element,

$$f_{l,l}^j(E) \equiv f_l^j(E) = \left(e^{2i\delta_l^j(E)} - 1\right)/2ik, \quad l = j\,,$$

where $\delta_l^j$ is the phase shift, a real function of $E$. But the waves $s=1, l=j\pm1$ are coupled, leading to a $2\times2$ matrix amplitude with the diagonal elements of the form [15],

$$f_{l,l}^j(E) = \left(e^{2i\delta_l^j(E)}\cos\epsilon_j(E) - 1\right)/2ik, \quad l = j\pm1\,,$$

where we have the mixing parameters $\epsilon_j(E)$ in addition to the phase shifts.

We may now evaluate the integral (3.2), using the phase shift analysis of the Nijmegen group [16]. Alternatively we may take advantage of the reconstruction of the full (Saclay) amplitudes from this analysis, also carried out by the same group. The general amplitude may be written in the c.m. frame as a $4\times4$ matrix in the Pauli basis as [17]

$$\mathcal{M}(\boldsymbol{p},\boldsymbol{p}') = \frac{1}{2}\{a_s + b_s + (a_s - b_s)\boldsymbol{\sigma}_1\cdot\boldsymbol{n}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{n} + (c_s + d_s)\boldsymbol{\sigma}_1\cdot\boldsymbol{m}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{m} + (c_s - d_s)\boldsymbol{\sigma}_1\cdot\boldsymbol{l}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{l}\}\,, \tag{3.4}$$

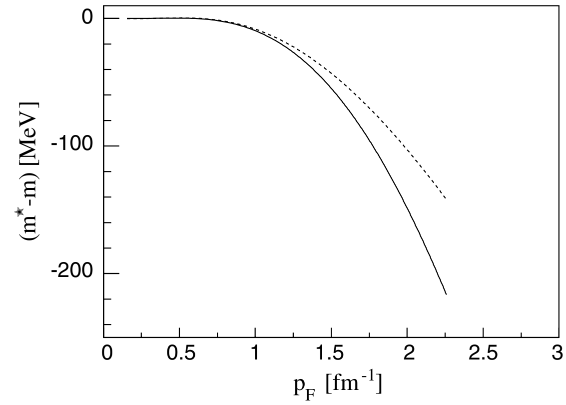


**Fig. 1.** Shift in nucleon mass in nuclear matter as a function of the Fermi momentum. The solid curve results from the partial-wave analysis of the Nijmegen group, while the dashed one is from the $s$-waves in the effective-range approximation.

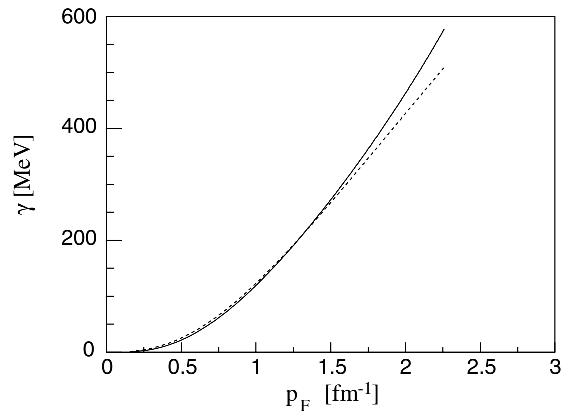


**Fig. 2.** Damping rate of nucleonic excitation in nuclear matter as a function of the Fermi momentum. The origin of solid and dashed curves are the same as in fig. 1.

where the Saclay amplitudes, $a_s$, $b_s$, $c_s$ and $d_s$, are complex functions of the energy and scattering angle. (We omit a fifth amplitude, which is zero in the forward direction.) Here $\boldsymbol{l}$, $\boldsymbol{m}$ and $\boldsymbol{n}$ are three mutually orthogonal unit vectors. The Pauli matrices $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$ act on the Pauli spinors $\chi$'s of the first and second nucleon. The spin-averaged, forward amplitude $\overline{f}$ is obtained from $\mathcal{M}$ as

$$\overline{f}(E) = \frac{1}{4}\sum_{\sigma_1,\sigma_2} \chi^\dagger_{\sigma_2}\chi^\dagger_{\sigma_1}\mathcal{M}(\boldsymbol{p},\boldsymbol{p})\chi_{\sigma_1}\chi_{\sigma_2} = \frac{1}{2}(a_s(E)+b_s(E)). \tag{3.5}$$

With the values of the Saclay amplitudes [16], we evaluate the real and the imaginary parts of the integral (3.2) at different Fermi momenta. The results are shown by the solid curves in figs. 1 and 2.

For an independent, but approximate, estimate, we also evaluate the integrals by including only the $s$-waves in the effective-range approximation. Here an $s$-wave amplitude is written as $f_0 = 1/(k\cot\delta - ik)$ with $k\cot\delta = -a^{-1}+rk^2/2$, where $a$ and $r$ are the scattering length and the effective range. The values of these constants are long known [13]: For the spin singlet state, $a=-23.7$, $r=2.7$ and for the spin triplet state, $a=5.39$, $r=1.70$, all in units of fm. This evaluation is shown by the dashed curves in figs. 1 and 2. It is seen that the higher partial waves contribute little up to about $p_F = 1\,\text{fm}^{-1}$.

## 4 Discussion

Here we have considered the nucleon self-energy in nuclear matter, in the limit of ignoring the effect of Pauli blocking on it. This self-energy can be expressed in terms of the forward spin-averaged $NN$ scattering amplitude in vacuum. We calculate its real and imaginary parts, using the phase shift analysis of experimental data on $NN$ scattering.

Our results may be compared with that of the self-consistent Hartree-Fock calclation [6] to get an idea of the importance of Pauli blocking in the Fermi medium. (Since we do not include the relevant part of the self-energy in the nucleon mass, it would be appropriate to compare our results with their so-called "non-relativistic" version of the results.) In both the calculations the mass shift is a strongly dependent function of the nuclear density. At normal nuclear density ($p_F = 1.35\,\text{fm}^{-1}$) they find the (real part of the) mass shift to be $-87$ MeV; in our calculation this value is attained at a higher density corresponding to $p_F = 1.70\,\text{fm}^{-1}$.

Our calculation, which is based only on experimental data, readily provides a check on the the original relativistic Brueckner calculation [6]. We just need redo this calculation without the Pauli operator in the equation for the reaction matrix. The resulting calculation with the phenomenological potential should yield the same functional dependence of the self-energy on nuclear density as we find here.

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